

Triple Integrals in Cylindrical Coordinates

Here we'll develop the animation for computing the volume of the paraboloid

$$z = 1 - x^2 - y^2$$

over the unit circle $r = 1$. We need to keep θ fixed while we generate the cross section

for $y = 0$ and then let θ vary from 0 to 2π

$$i := \begin{cases} 0 & \text{if } \text{FRAME} \leq 20 \\ (0.. \text{FRAME} - 20) & \text{otherwise} \end{cases}$$

For the first 20 frames θ is fixed to be 0 then varies from 0 to 2 in increments of $\pi/24$. This is standard for almost every case (as with polar coordinates).

$$\theta_i := \pi \cdot \frac{i}{24}$$

$$j := 0..10$$

This allows r to vary from 0 to 2 in increments of .1. The range on r is dependent on the particular problem.

$$r_j := \frac{j}{10}$$

Axes:

$$i := 0..20 \quad X_{i,1} := 0 \quad Y_{i,1} := 0 \quad Z_{i,1} := (1 - 10) \cdot 1$$

The paraboloid

$$X_{i,j} := (r_j) \cdot \cos(\theta_i) \quad Y_{i,j} := (r_j) \cdot \sin(\theta_i) \quad Z_{i,j} := \left[1 - (X_{i,j})^2 - (Y_{i,j})^2 \right]$$

The following 6 sets of parametric equations give the line segments from the plane to the paraboloid for $y = 0$, creating the cross section

$$s := 0.. \text{FRAME} \quad t(s) := s \cdot .25 \quad w := 0..1$$

$$x_{w,s} := 0 \quad y_{w,s} := 0 \quad z_{w,s} := t(s)$$

$$x_{1,w,s} := \begin{cases} .1 & \text{if } \text{FRAME} \leq 10 \\ .2 & \text{otherwise} \end{cases} \quad y_{1,w,s} := 0$$

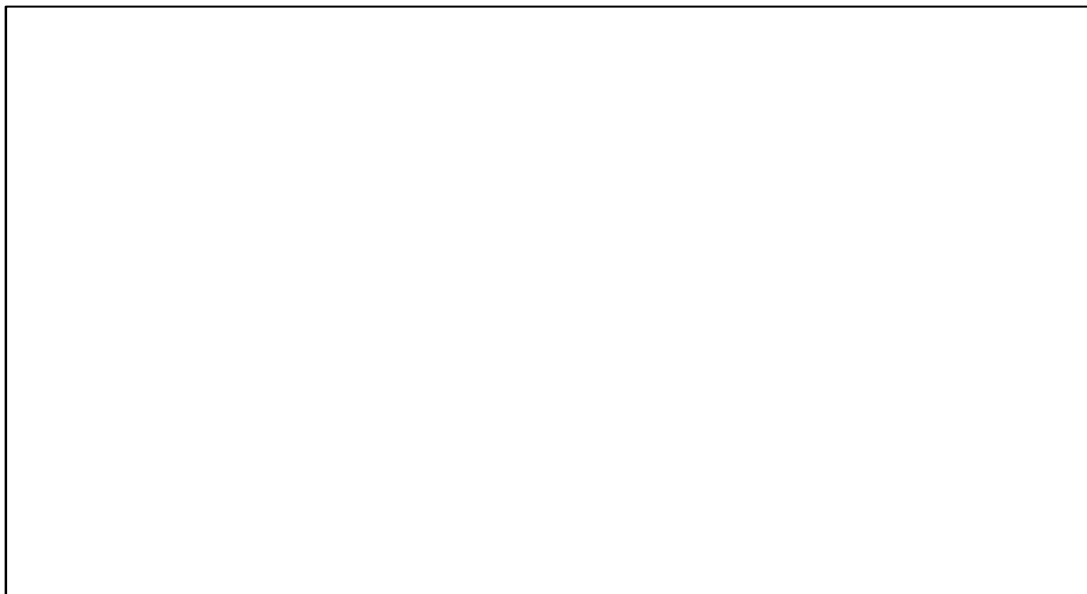
$$z_{1,w,s} := \begin{cases} t(s) & \text{if } \text{FRAME} \leq 10 \\ \frac{.96}{.5} \cdot (t(s) - 2.5) & \text{if } 10 < \text{FRAME} \leq 12 \\ \frac{.96}{.5} \cdot [t(s) - 2.5 - .25 \cdot (\text{FRAME} - 12)] & \text{otherwise} \end{cases}$$

$$x2_{w,s} := \begin{cases} .1 & \text{if FRAME} \leq 12 \\ .4 & \text{otherwise} \end{cases} \quad y2_{w,s} := 0 \quad z2_{w,s} := \begin{cases} t(s) & \text{if FRAME} \leq 12 \\ \frac{.84}{.5} \cdot (t(s) - 3) & \text{if } 12 < \text{FRAME} \leq 14 \\ \frac{.84}{.5} \cdot [t(s) - 3 - .25 \cdot (\text{FRAME} - 14)] & \text{otherwise} \end{cases}$$

$$x3_{w,s} := \begin{cases} .1 & \text{if FRAME} \leq 14 \\ .6 & \text{otherwise} \end{cases} \quad y3_{w,s} := 0 \quad z3_{w,s} := \begin{cases} t(s) & \text{if FRAME} \leq 14 \\ \frac{.64}{.5} \cdot (t(s) - 3.5) & \text{if } 14 < \text{FRAME} \leq 16 \\ \frac{.64}{.5} \cdot [t(s) - 3.5 - .25 \cdot (\text{FRAME} - 16)] & \text{otherwise} \end{cases}$$

$$x4_{w,s} := \begin{cases} .1 & \text{if FRAME} \leq 16 \\ .8 & \text{otherwise} \end{cases} \quad y4_{w,s} := 0 \quad z4_{w,s} := \begin{cases} t(s) & \text{if FRAME} \leq 16 \\ \frac{.36}{.5} \cdot (t(s) - 4) & \text{if } 16 < \text{FRAME} \leq 18 \\ \frac{.36}{.5} \cdot [t(s) - 4 - .25 \cdot (\text{FRAME} - 18)] & \text{otherwise} \end{cases}$$

$$x5_{w,s} := \begin{cases} .1 & \text{if FRAME} \leq 18 \\ .9 & \text{otherwise} \end{cases} \quad y5_{w,s} := 0 \quad z5_{w,s} := \begin{cases} t(s) & \text{if FRAME} \leq 18 \\ \frac{.19}{.5} \cdot (t(s) - 4.5) & \text{if } 18 < \text{FRAME} \leq 20 \\ \frac{.19}{.5} \cdot [t(s) - 4.5 - .25 \cdot (\text{FRAME} - 20)] & \text{otherwise} \end{cases}$$



$(X, Y, Z), (Y1, Z1, X1), (Z1, X1, Y1), (x, y, z), (x1, y1, z1), (x2, y2, z2), (x3, y3, z3), (x4, y4,$