

Summary -Observations and Terminology

The following are based on the animation $f(x)$ and $f'(x)$ but apply to all differentiable functions in general.

Observation 1

$f(x)$ is increasing on an open interval if and only if $f'(x)$ is positive on that interval

Note f is increasing on the intervals $(0,1)$, $(3,5)$ and $(5,7)$ precisely the intervals where $f'(x)$ is positive.

Observation 2

$f(x)$ is decreasing on an open interval if and only if $f'(x)$ is negative on that interval .

Note f is decreasing on the interval $(3,5)$.

Observation 3

$f(x)$ has a horizontal tangent line where $f'(x) = 0$. $f(x)$ has horizontal tangents at $x = 1, 3, \text{and } 5$.

Terminology 1

If $f'(c) = 0$ we say $f(x)$ has a critical point at $x = c$. Another term for a critical point is stationary point.

Terminology 2

If $f(c) \geq f(x)$ for all x in some open interval containing c we say $f(x)$ has a local maximum at $x = c$. $f(c)$ is called the local maximum value.

Terminology 3

If $f(c) \leq f(x)$ for all x in some open interval containing c we say $f(x)$ has a local minimum at $x = c$. $f(c)$ is called the local minimum value.

Terminology 4

If $f(c) \geq f(x)$ for all x then we say $f(x)$ has an absolute or global maximum at $x = c$. Sometimes if we just say maximum we mean global or absolute maximum. $f(c)$ is called the absolute maximum value.

Terminology 5

If $f(c) \leq f(x)$ for all x then we say $f(x)$ has an absolute or global minimum at $x = c$.
Sometimes if we just say minimum we mean global or absolute minimum.
 $f(c)$ is called the absolute minimum value.

Terminology 6

Either a local minimum or local maximum is referred to as an extremum

<u>singular</u>	<u>plural</u>
maximum	maxima
minimum	minima
extremum	extrema

Observation 4

If $f(x)$ is differentiable then the local extrema occur at the critical points . Note $f(x)$ has a local maximum at the critical point $x = 1$ and a local minimum at the critical point $x = 3$.

Be Careful 1

Not every critical point is an extremum as is evidenced by $f(x)$ at $x = 5$.

Observation 5

If $f'(x)$ changes from positive to negative at a critical point $x = c$ then $f(x)$ has a local maximum at $x = c$ as is evidenced at $x = 1$.

Observation 6

If $f'(x)$ changes from negative to positive at a critical point $x = c$ then $f(x)$ has a local minimum at $x = c$ as is evidenced at $x = 3$.

Observation 7

If $f'(x)$ doesn't change sign at a critical point $x = c$ $f(x)$ has an inflection point at $x = c$ i.e. a point where the concavity of $f(x)$ changes.

Further Details

1. Extreme Value Theorem

If $f(x)$ is continuous on the closed interval $[a,b]$ it has both a maximum and a minimum.

Observation 8

If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b) then the extrema occur either at the endpoints or at critical points.

2. If $f(x)$ is not differentiable then a local extremum can occur where the derivative does not exist.

For example for $f(x) = |x|$ $f(x)$ is not differentiable at $x = 0$ but there is a local (in fact global) minimum at $x = 0$.

See the download [A Non-Differentiable Example](#) for the details

Terminology 7

If $f(x)$ is continuous at $x = c$ but not differentiable we say $f(x)$ has a cusp at $x = c$.

Two examples are $f(x) = |x|$ at $x = 0$ and $f(x) = x^{2/3}$ at $x = 0$.