

Formatting Animations for Riemann Sums

Right Sums

In the example on the site we used Riemann Sums to approximate $\int_1^2 x^2 dx$

`x := 1, 1.001..2` Sets up the interval along the x -axis

`f(x) := x^2` The function in question

`n := 2 + FRAME` sets up the number of subintervals. I use n+2 so the initial image already has 2 rectangles

We'll need the following information:

1. The width of each sub interval $\Delta x = \frac{b - a}{n}$ In our example $\Delta x = \frac{1}{n}$

2, Define the partition points $x_k = a + \frac{k \cdot (b - a)}{n}$ In our example $x_k = 1 + \frac{k}{n}$

3. Compute $f(x_k)$ the height of each rectangle over each sub interval $f(x_k) = \left(1 + \frac{k}{n}\right)^2$

4. The right Riemann sum -- $\sum_{k=1}^n (f(x_k) \cdot \Delta x)$ In our example

$$\sum_{k=1}^n \left[\left(1 + \frac{k}{n}\right)^2 \cdot \frac{1}{n} \right]$$

5. Define $fR(x) = f(a + \Delta x) + \sum_{k=1}^n [\Phi(x - x_k) \cdot (f(x_{k+1}) - f(x_k))]$ to graph the rectangles

$$fR(x) := \left(1 + \frac{1}{n}\right)^2 + \sum_{k=1}^n \left[\Phi \left[x - \left(1 + \frac{k}{n}\right) \right] \cdot \left[\left(1 + \frac{k+1}{n}\right)^2 - \left(1 + \frac{k}{n}\right)^2 \right] \right]$$

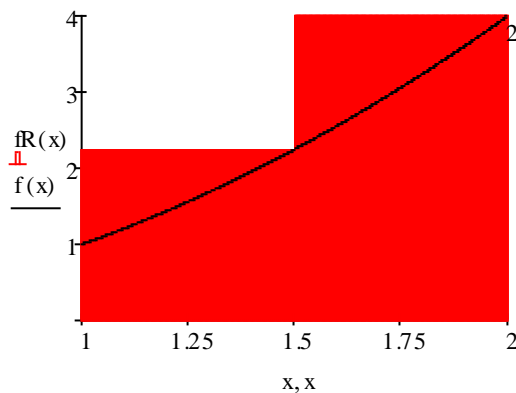
The actual format looks like:

$$x := 1, 1.001, 2 \quad n := 2 + \text{FRAMI} \quad f(x) := x^2$$

$$fR(x) := \left(1 + \frac{1}{n}\right)^2 + \sum_{k=1}^n \left[\Phi \left[x - \left(1 + \frac{k}{n}\right) \right] \cdot \left[\left(1 + \frac{k+1}{n}\right)^2 - \left(1 + \frac{k}{n}\right)^2 \right] \right]$$

On the Graph for Trace 1 change from lines to bars under TYPE menu

$$Rn = \sum_{k=1}^n \left[\left(1 + \frac{k}{n}\right)^2 \cdot \frac{1}{n} \right] = 3.125$$



The Left sum is almost identical. The only differences are:

$$1. \quad \ln fL(x) = f(a) + \sum_{k=1}^n \left[\Phi(x - x_{k+1}) \cdot (f(x_{k+1}) - f(x_k)) \right]$$

i.e. we use $f(a)$ instead of $f(a+\Delta x)$, sum from 0 to $n-1$ instead of 1 to n and replace $\Phi(x - x_k)$ with $\Phi(x - x_{k+1})$

$$2. \quad \text{For } Ln = \sum_{k=0}^{n-1} \left[\left(1 + \frac{k}{n}\right)^2 \cdot \frac{1}{n} \right] \quad \text{we again sum from 0 to } n-1$$

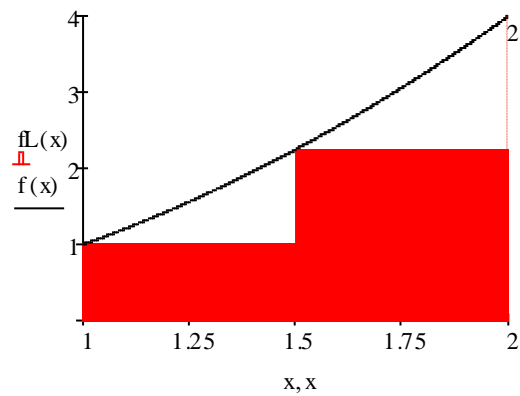
$$x := 1, 1.001, 2$$

$$n := 2 + \text{FRAMI}$$

$$f(x) := x^2$$

$$fL(x) := (1)^2 + \sum_{k=0}^{n-1} \left[\Phi \left[x - \left(1 + \frac{k+1}{n} \right) \right] \cdot \left[\left(1 + \frac{k+1}{n} \right)^2 - \left(1 + \frac{k}{n} \right)^2 \right] \right]$$

$$Ln = \sum_{k=0}^{n-1} \left[\left(1 + \frac{k}{n} \right)^2 \cdot \frac{1}{n} \right] = 1.625$$



The Midpoint Rule

The midpoint of each subinterval is $c_k = \frac{x_k + x_{k-1}}{2}$

In our example

$$c_k = \frac{1 + \frac{k}{n} + \left(1 + \frac{k-1}{n}\right)}{2} = \frac{2k + 2n - 1}{2 \cdot n}$$

$$M_n = \sum_{k=1}^n (f(c_k) \cdot \Delta x) = \sum_{k=1}^n \left[\left(\frac{2k + 2n - 1}{2 \cdot n} \right)^2 \cdot \frac{1}{n} \right]$$

$$fM(x) = \left(1 + \frac{\Delta x}{2}\right) + \sum_{k=1}^n [\Phi(x - x_k) \cdot (f(c_{k+1}) - f(c_k))]$$

$$fM(x) := \left(1 + \frac{1}{2n}\right)^2 + \sum_{k=1}^n \left[\Phi \left[x - \left(1 + \frac{k}{n}\right) \right] \cdot \left[\left(\frac{2(k+1) + 2n - 1}{2 \cdot n} \right)^2 - \left(\frac{2k + 2n - 1}{2 \cdot n} \right)^2 \right] \right]$$

$$M_n = \sum_{k=1}^n \left[\left(\frac{2k + 2n - 1}{2 \cdot n} \right)^2 \cdot \frac{1}{n} \right] = 2.313$$

