Partial Derivatives

To illustrate we'll consider the partial derivatives of the parabaloid $f(x,y) = 1 - x^2 - y^2$ at the point

(1/3,1/3,7/9).

Our first set up will be for δ f/δ y

We start by defining the surface using the cylindrical coordinate format:

$i := 0..48 \quad \theta_i := \pi \cdot \frac{i}{24} \quad j := 0..10 \quad r_j := \frac{j}{10}$

$X_{i,j} := r_j \cdot \cos(\theta_i) \quad Y_{i,j} := r_j \cdot \sin(\theta_i) \quad Z_{i,j} := 1 - (X_{i,j})^2 - (Y_{i,j})^2$

For Plot 1 under the Appearance Tab change the weight to 0.1

Now we define the point

$\tau := 0..1$

$a_{i,\tau} := \frac{1}{3} \quad b_{i,\tau} := \frac{1}{3} \quad c_{i,\tau} := \frac{7}{9}$

For Plot 2 under the General Tab change from surface plot to Data Points and under the Appearance Tab change the weight to 3

Now we define the curve of intersection of the surface with plane x = 1/3

$m := 0..100 \quad w(m) := .01 \cdot m$

$x_{\tau,m} := \frac{1}{3} \quad y_{\tau,m} := w(m) \quad z_{\tau,m} := 1 - (x_{\tau,m})^2 - (y_{\tau,m})^2$

For Plot 3 under the Appearance Tab change the weight to 2.

Now we define the secant line through (1/3,1/3,7/9).

$h := .5 - .02 \cdot FRAME$

(h is the interval over which the average rate of change is calculated as in the case of the secant line for functions of a single variable)

We can use $x_{i,m}$ and $y_{i,m}$ from the curve of intersection all we have to do is to define a new z value

$\frac{\delta f}{\delta y} = -2 \cdot y$ so when y =1/3 we have $\frac{\delta f}{\delta y} = \frac{-2}{3}$ so $z = \frac{\delta f}{\delta y}\left(\frac{1}{3}, \frac{1}{3} + h\right)\left(y - \frac{1}{3}\right) + \frac{7}{9}$

We define:

$$z1_{\tau, m} := \left(\frac{-2}{3} - h\right) \cdot \left(\left(y_{\tau, m} - \frac{1}{3}\right)\right) + \frac{7}{9}$$

For Plot 4 under the Appearance Tab change the weight to 2.

The actual format is then :

$i := 0 .. 48 \quad \theta_i := \pi \cdot \frac{i}{24} \quad j := 0 .. 10 \quad r_j := \frac{j}{10}$

$X_{i,j} := r_j \cdot \cos(\theta_i) \quad Y_{i,j} := r_j \cdot \sin(\theta_i) \quad Z_{i,j} := 1 - (X_{i,j})^2 - (Y_{i,j})^2$

$\tau := 0 .. 1$

$a_{i,\tau} := \frac{1}{3} \quad b_{i,\tau} := \frac{1}{3} \quad c_{i,\tau} := \frac{7}{9}$

$m := 0 .. 100 \quad w(m) := .01 \cdot m$

$x_{\tau, m} := \frac{1}{3} \quad y_{\tau, m} := w(m) \quad z_{\tau, m} := 1 - (x_{\tau, m})^2 - (y_{\tau, m})^2$

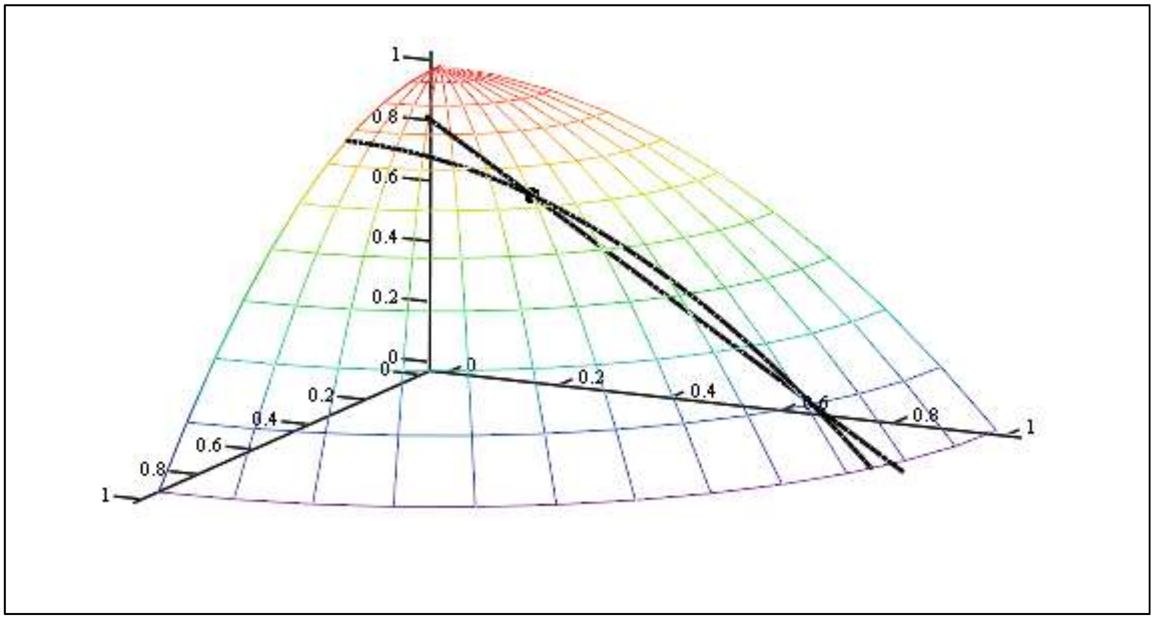
$h := .5 - .02 \cdot FRAME$

$$z1_{\tau, m} := \left(\frac{-2}{3} - h\right) \cdot \left(\left(y_{\tau, m} - \frac{1}{3}\right)\right) + \frac{7}{9}$$

Use 24 frames.

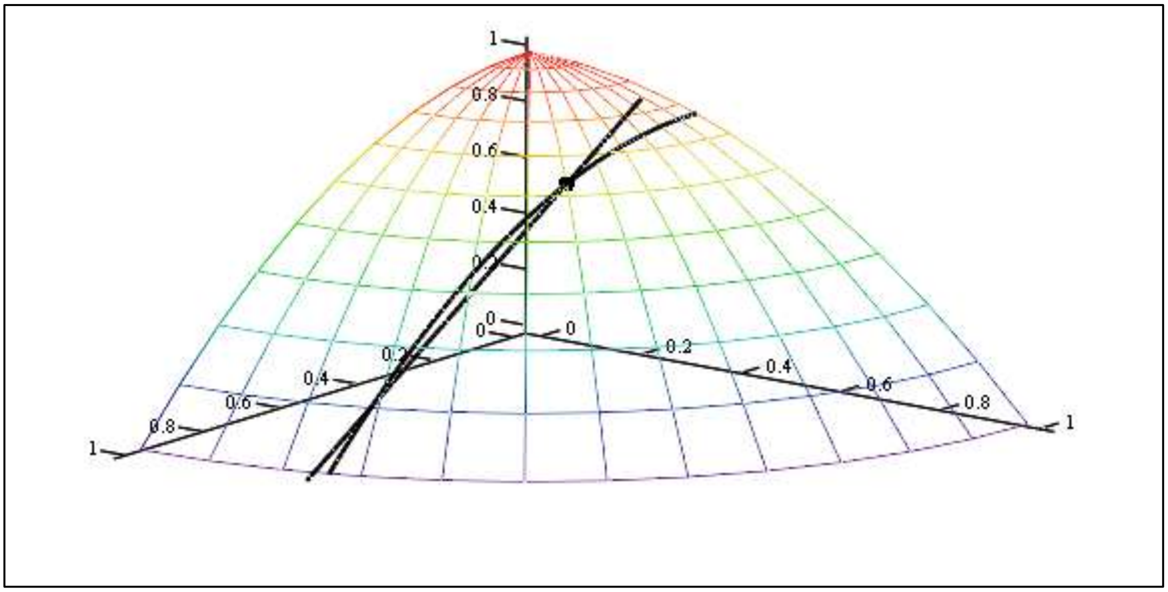
Estimate Exact

$\mathbf{h} = \mathbf{0.5}$ $\left(\frac{-2}{3} - h\right) = -1.167$ $\frac{-2}{3}$



$(X, Y, Z), (a, b, c), (x, y, z), (x, y, z1)$

For the partial derivative with respect to x all we have to do is interchange x and y



$(X, Y, Z), (a, b, c), (y, x, z), (y, x, z1)$