

Format for incoming and outgoing light rays reflected off a parabolic mirror

$$f(x) := \frac{x^2}{4} \quad \text{defines the equation of the mirror with focal point at } (0,1)$$

So now we define the equations of the incoming light rays.

$t := 0, .1..2$  Later to animate we'll change 2 to FRAME/10 but for now we'll use 2 to show on the graph the situation.

In the x direction we want vertical lines, i.e. a constant value of x, until the light rays hit the mirror. Once they hit the mirror at  $t = 1$  then the light ray goes from the x coordinate of the mirror to 0 between  $t = 1$  and  $t = 2$ .

For example at  $x = 1$

$$X(t) = 1 + \Phi(t - 1) \cdot (1 - t).$$

at  $x = 2$   $X(t) = 2 + \Phi(t - 1) \cdot (2 - 2t)$  and so on.

We don't have to define several functions we only need one:

$$X(t, n) := n + \Phi(t - 1) \cdot [n \cdot (1 - t)]$$

In the y direction we start all of the light rays at  $y = 4$ . Then y decreases until the light ray hits the mirror at  $t = 1$  then goes from the y-coordinate of the mirror to the y-coordinate of the focal point i.e. 1 between  $t = 1$  and  $t = 2$ .

For example when  $x = 1$  we have the light ray going from 4 to 1/4 at  $t = 1$  to  $y = 1$  at  $t = 2$ .

$$Y(t) = 4 + \left(\frac{1}{4} - 4\right) \cdot t + \Phi(t - 1) \cdot \left[\frac{1}{4} + \left(1 - \frac{1}{4}\right) \cdot (t - 1) - \left[4 + \left(\frac{1}{4} - 4\right) \cdot t\right]\right]$$

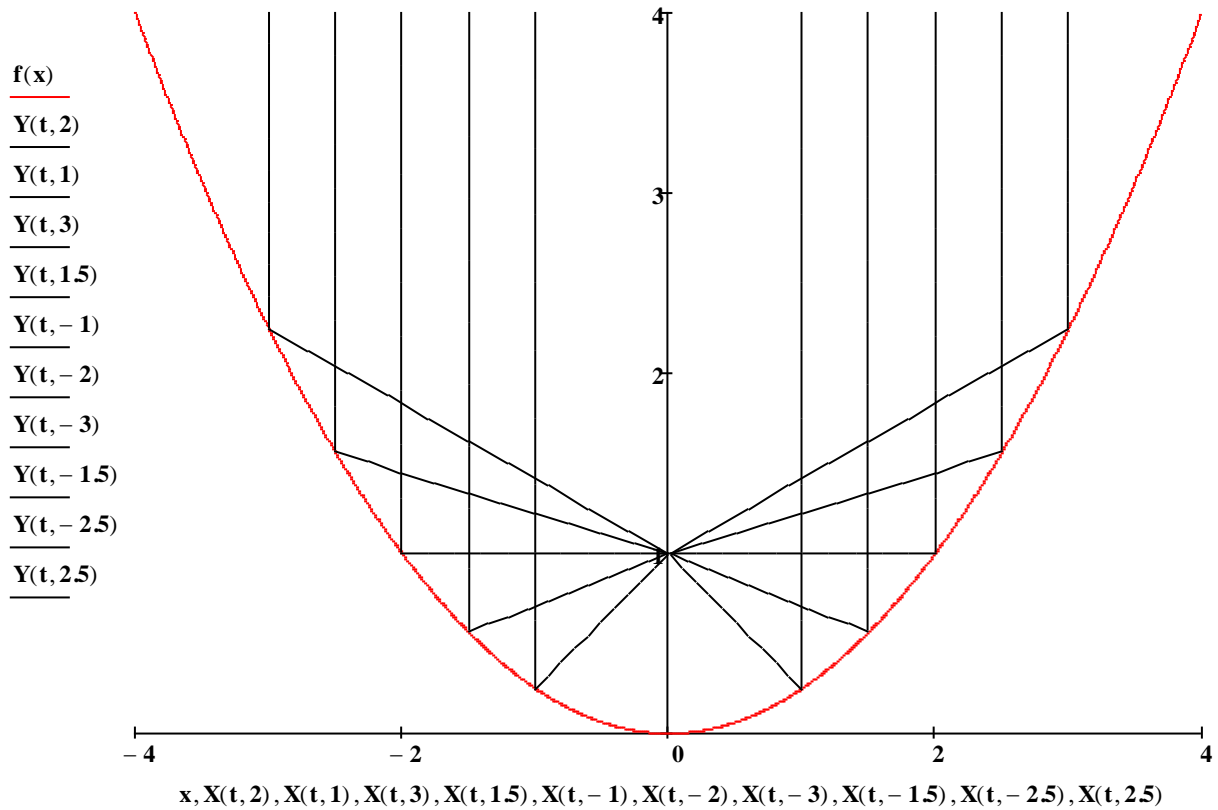
when  $x = 2$

$$Y(t) = \left[4 + \left(\frac{2^2}{4} - 4\right) \cdot t\right] + \Phi(t - 1) \cdot \left[\frac{2^2}{4} + \left(1 - \frac{2^2}{4}\right) \cdot (t - 1) - \left[4 + \left(\frac{2^2}{4} - 4\right) \cdot t\right]\right]$$

Again we can define a single function:

$$Y(t, n) := \left[4 + \left(\frac{n^2}{4} - 4\right) \cdot t\right] + \Phi(t - 1) \cdot \left[\frac{n^2}{4} + \left(1 - \frac{n^2}{4}\right) \cdot (t - 1) - \left[4 + \left(\frac{n^2}{4} - 4\right) \cdot t\right]\right]$$

The graph would then be set up as :



To set up the animation then we would have:

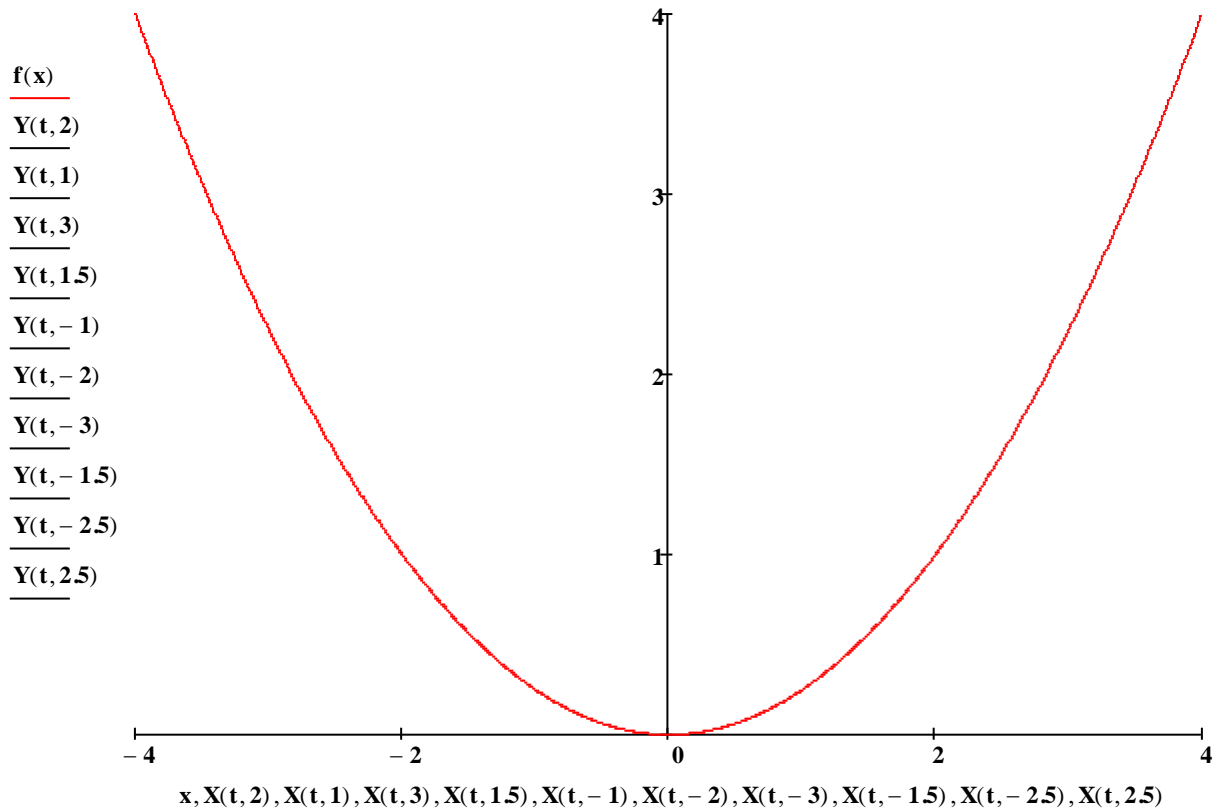
$$\underline{\underline{f(x)}} := \frac{x^2}{4}$$

$$t := 0, .1.. \frac{\text{FRAME}}{10}$$

$$\underline{\underline{X(t, n)}} := n + \Phi(t - 1) \cdot [n \cdot (1 - t)]$$

$$\underline{\underline{Y(t, n)}} := \left[ 4 + \left( \frac{n^2}{4} - 4 \right) \cdot t \right] + \Phi(t - 1) \left[ \frac{n^2}{4} + \left( 1 - \frac{n^2}{4} \right) \cdot (t - 1) - \left[ 4 + \left( \frac{n^2}{4} - 4 \right) \cdot t \right] \right]$$

We would use 20 frames.

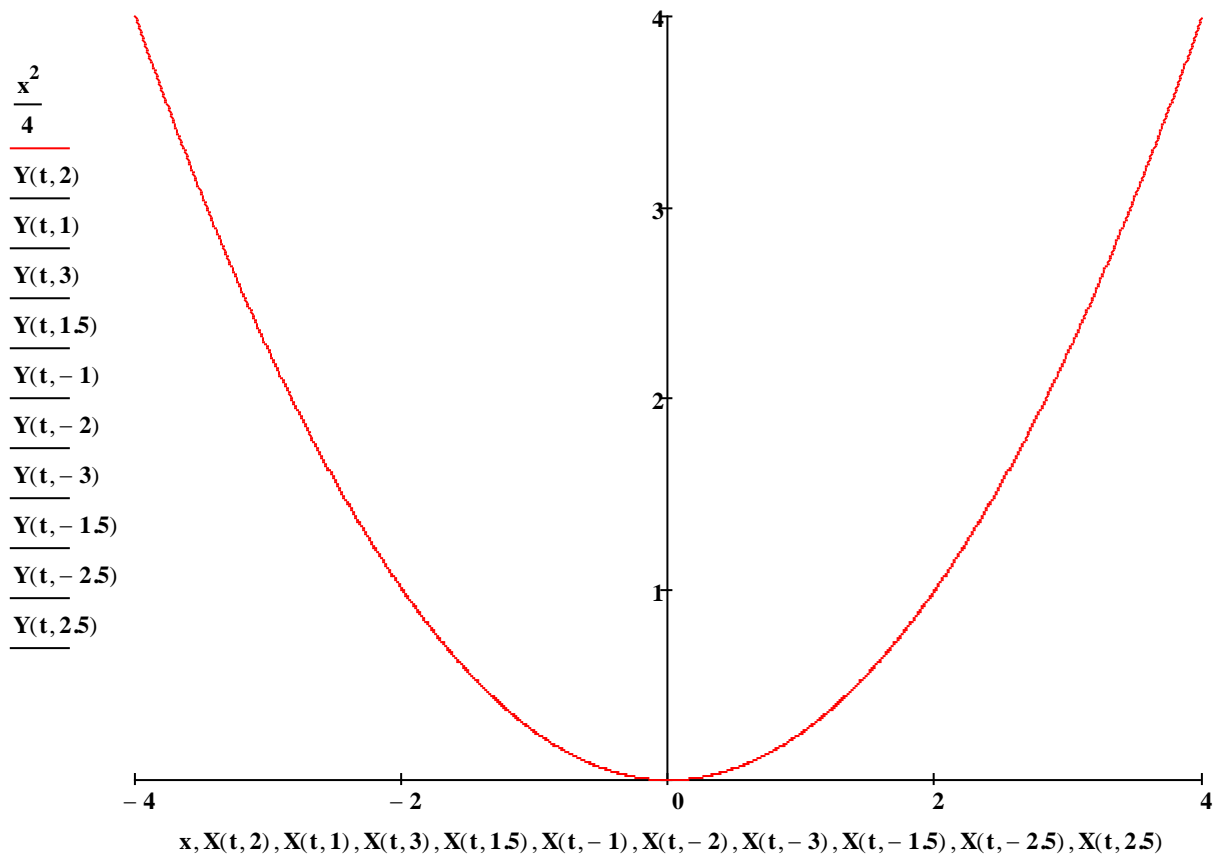


For outgoing light rays emanating from the focal point then being reflected from the mirror vertically we have

$$t := 0, .1.. \frac{\text{FRAME}}{10}$$

$$\underline{\underline{X}}(t, n) := n \cdot t + \Phi(t - 1) \cdot (n - n \cdot t)$$

$$\underline{\underline{Y}}(t, n) := \left[ \left[ 1 + \left( \frac{n^2}{4} - 1 \right) \cdot t \right] + \Phi(t - 1) \left[ \frac{n^2}{4} + \left( 1 - \frac{n^2}{4} \right) \cdot (t - 1) - \left[ 4 + \left( \frac{n^2}{4} - 4 \right) \cdot t \right] \right] \right]$$



Incoming  $t := 0, .1.. \frac{\text{FRAME}}{10}$

$\underline{X}(t, n) := n + \Phi(t - 1) \cdot [-n \cdot (t - 1)]$

$$\mathbf{Y(t, n)} := \left[ \left[ 5 + \left( \frac{n^2}{4} - 5 \right) \cdot t \right] + \Phi(t-1) \left[ \frac{n^2}{4} + \left( 1 - \frac{n^2}{4} \right) \cdot (t-1) - \left[ 5 + \left( \frac{n^2}{4} - 5 \right) \cdot t \right] \right] \right]$$

