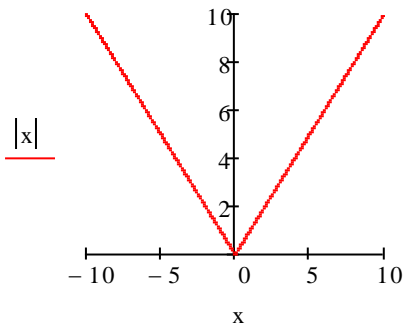


A Point of Non-Differentiability

Consider $f(x) = |x|$.



Let's Calculate the derivative at $x = 0$.

$$\text{by definition: } f'(0) = \lim_{x \rightarrow 0} \frac{(f(x) - f(0))}{x - 0}$$

Recal a limit and hence the derivative only exists if the right and left hand limits are equal

So we will consider the right and left hand limits.

$$\lim_{x \rightarrow 0^+} \frac{(f(x) - f(0))}{x - 0} = \lim_{x \rightarrow 0^+} \frac{|x|}{x} = \lim_{x \rightarrow 0^+} \frac{x}{x} = 1 \quad \text{since } |x| = x \text{ if } x > 0$$

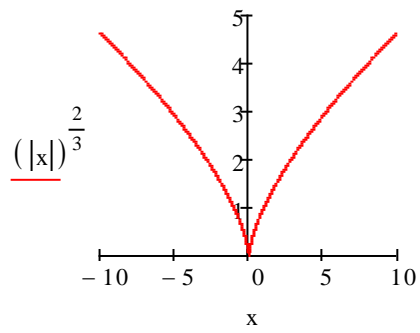
$$\lim_{x \rightarrow 0^-} \frac{(f(x) - f(0))}{x - 0} = \lim_{x \rightarrow 0^-} \frac{|x|}{x} = \lim_{x \rightarrow 0^-} \frac{-x}{x} = -1 \quad \text{since } |x| = -x \text{ if } x < 0.$$

Therefore $\lim_{x \rightarrow 0} \frac{(f(x) - f(0))}{x - 0}$ **does not exist.**

Therefore $f(x)$ has no derivative at $x = 0$ and we say $f(x)$ is non-differentiable at $x = 0$.

If $f(x)$ is continuous at a point but non-differentiable at that point we say $f(x)$ has a cusp at that point.

As another example consider $f(x) = x^{2/3}$ at $x = 0$.



Do the Calculation or observe the right hand limit is +infinity and the left hand limit is – infinity.
The point is the limit and therefore the derivative does not exist.