

Rigorous Limit Format

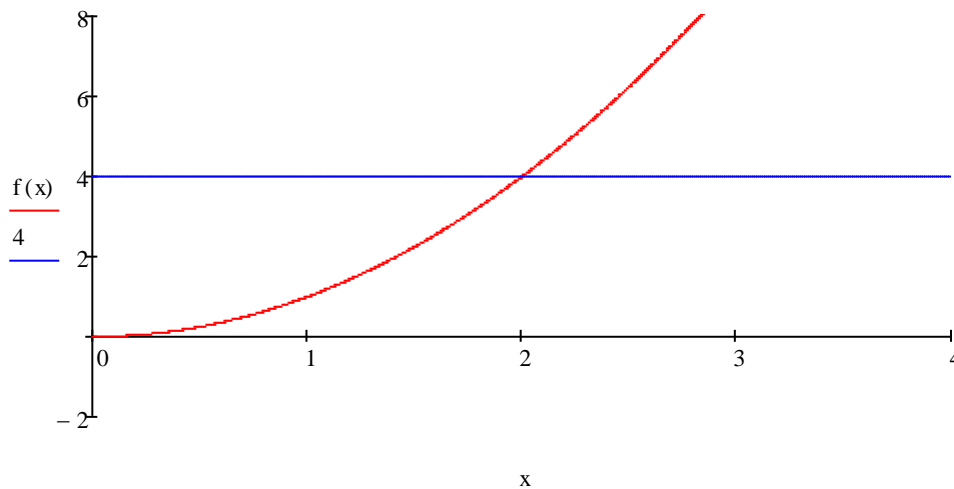
$f(x) := x^2$ Defines $f(x)$

$\varepsilon := 1.5 - .05 \text{FRAME}$ allows ε to decrease in increments .05

In setting the number of frames we will use 29 frames since $1.5 - .05 \cdot 29 = .05$

$\delta := \min\left(\frac{\varepsilon}{5}, 1\right)$ Defines δ (see the notes on Calculus7.com on why $\frac{\varepsilon}{5}$ is used)

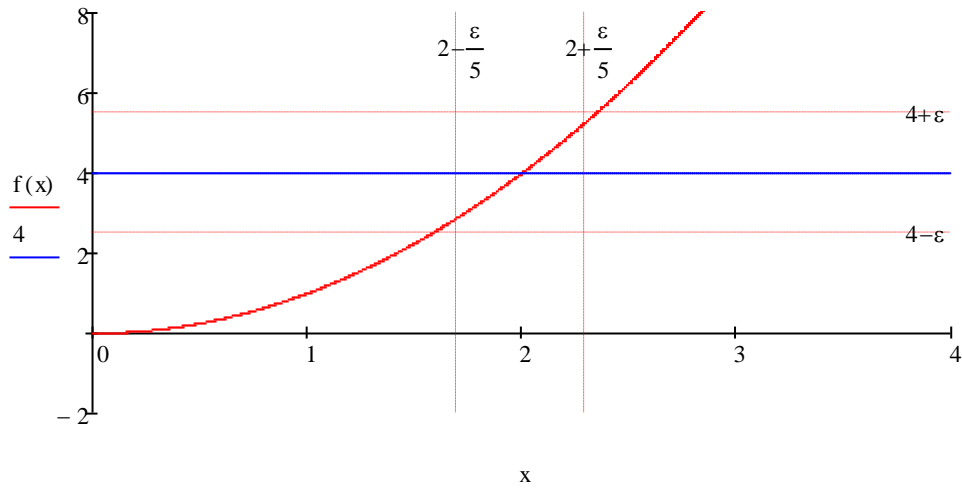
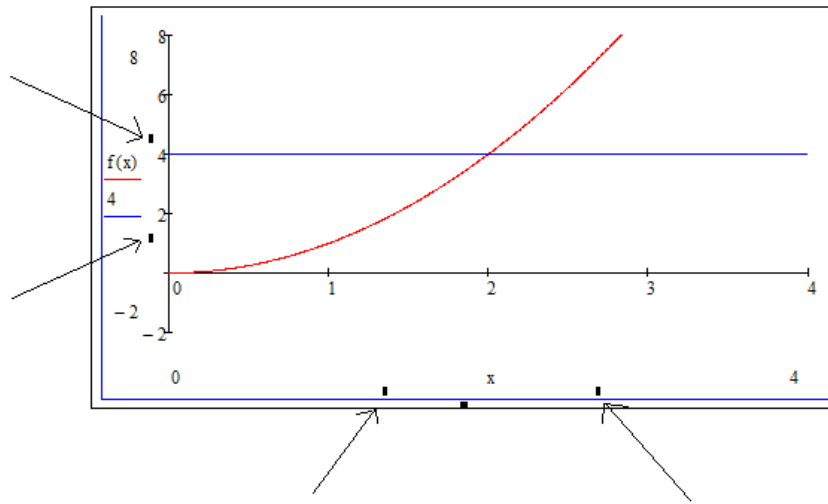
In this Animation we use the SHOW MARKERS.
Start by graphing $f(x)$ and 4 on the vertical.



In the format window under x- y axes click on SHOW MARKERS for both x and y.

On the graph you will then see place holders. On the vertical put $4 - \varepsilon$ and $4 + \varepsilon$

and on the horizontal $2 - \frac{\varepsilon}{5}$ and $2 + \frac{\varepsilon}{5}$.



Now animate using 29 frames--Here I used 2 Frames/sec but as always this is the decision of the animator.

Rigorous Limits at Infinity

Again we will use the show markers

$$f(x) := 1 - \frac{\sin(x)}{x} \quad \text{Here the limit is 1 so on the vertical we use } 1 - \varepsilon \text{ and } 1 + \varepsilon$$

But what about M?

$$|f(x) - 1| = \left| 1 - \frac{\sin(x)}{x} - 1 \right| = \left| \frac{\sin(x)}{x} \right| < \frac{1}{x} \quad \text{if } x > \frac{1}{\varepsilon} \quad \text{then } |f(x) - 1| < \varepsilon$$

so if $M = \frac{1}{\varepsilon}$ then if $x > M$ $|f(x) - 1| < \varepsilon$. However here I think it instructive

to use over kill and I use $M := \frac{2}{\varepsilon}$ which is what I use in the SHOW MARKER placeholder on the horizontal.

$$\varepsilon := .1 - .001 \text{FRAME} \quad M := \frac{2}{\varepsilon}$$

I set the limits on the y axes to go from .8 to 1.2 and on the x axes 1 to 120.

I want to stop the animation at $x = 100$ -- this is of course arbitrary -large enough to illustrate the principal but not so large the graphics become unreadable.

I use 80 Frames because $\frac{2}{\varepsilon} = 100$ yields $\varepsilon = .02$ and solving $.1 - .001 \text{FRAME} = .02$ yields $\text{FRAME} = 80$.

I would recommend 3Frames/sec animation speed.

