Directional Derivative Demonstration

Again we will be using the parabaloid $z = 1 - x^2 - y^2$ with domain the unit circle r = 1 in the first quadrant.

The idea we want to get across here is that the rate at which z changes at a point depends on the direction one travels from that point in the domain. This is used as a motivation for development of the directional derivative.

Here we will work in the first octant so we'll not put the axes on the graph. We'll consider the pt (1/2,1/2,0) in the domain and correspondingly the pt (1/2,1/2,1/2) on the surface.

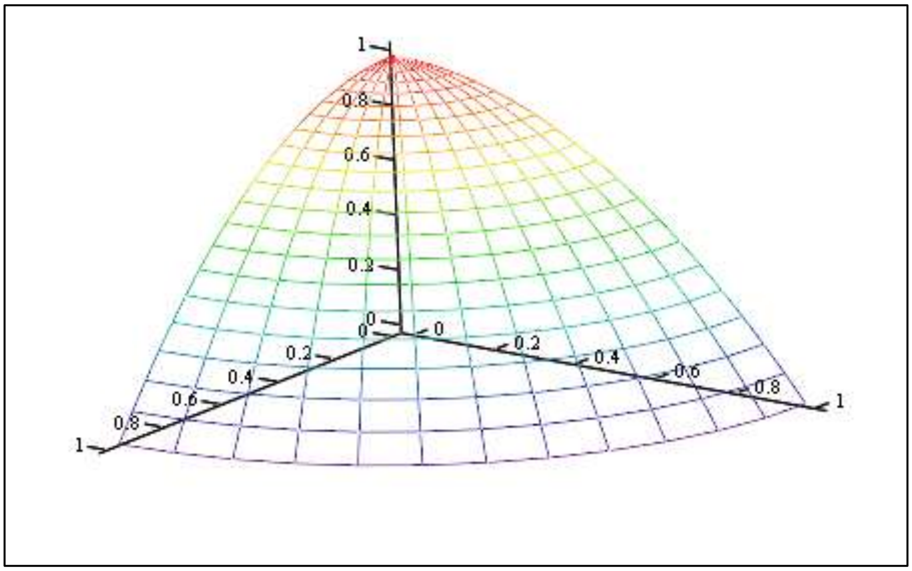
We start by defining the surface

$i := 0..12 \quad \theta_i := \pi \cdot \frac{i}{24} \qquad j := 0..20 \qquad r_j := \frac{j}{20}$

$X_{i,j} := r_j \cdot \cos(\theta_i) \quad Y_{i,j} := r_j \cdot \sin(\theta_i) \quad Z_{i,j} := 1 - (X_{i,j})^2 - (Y_{i,j})^2$

for Plot 1 change weight to .5 to lighten the surface.

Under General in the Format window under Axes Style choose Corner



(X, Y, Z)

Now we define the point (1/2,1/2,1/2) on the surface and correspondingly the point (1/2,1/2,0) in the plane.

$s := 0..FRAME \quad t(s) := s \cdot .02 \quad \tau := 0..1 \quad s1 := FRAME$

Point on the surface $x_{\tau,s} := \frac{1}{2} \quad y_{\tau,s} := \frac{1}{2} \quad z_{\tau,s} := \frac{1}{2}$ for Plot 2 under Appearance click on Draw points and change the size to 2.

Point in the plane $x1_{\tau,s} := \frac{1}{2} \quad y1_{\tau,s} := \frac{1}{2} \quad z1_{\tau,s} := 0$ for Plot3 under Appearance click on Draw points and change the size to 2.

We define the directional vectors in the plane and correspondingly the curve on the surface corresponding to each vector

In the direction of the gradient

In the plane

$$x2_{\tau,s} := \frac{1}{2} - t(s) \quad y2_{\tau,s} := \frac{1}{2} - t(s) \quad z2_{\tau,s} := 0$$

On the surface

$$x3_{\tau,s} := \frac{1}{2} - t(s) \quad y3_{\tau,s} := \frac{1}{2} - t(s) \quad z3_{\tau,s} := 1 - \left(x3_{\tau,s}\right)^2 - \left(y3_{\tau,s}\right)^2$$

In the i direction in the plane

$$x4_{\tau,s} := \frac{1}{2} + t(s) \quad y4_{\tau,s} := \frac{1}{2} \quad z4_{\tau,s} := 0$$ Change the color to red

In the i - direction on the surface

$$x5_{\tau,s} := \frac{1}{2} + t(s) \quad y5_{\tau,s} := \frac{1}{2} \quad z5_{\tau,s} := 1 - \left(x5_{\tau,s}\right)^2 - \left(y5_{\tau,s}\right)^2$$ Change the color to red

In the $-\vec{j}$ direction in the plane

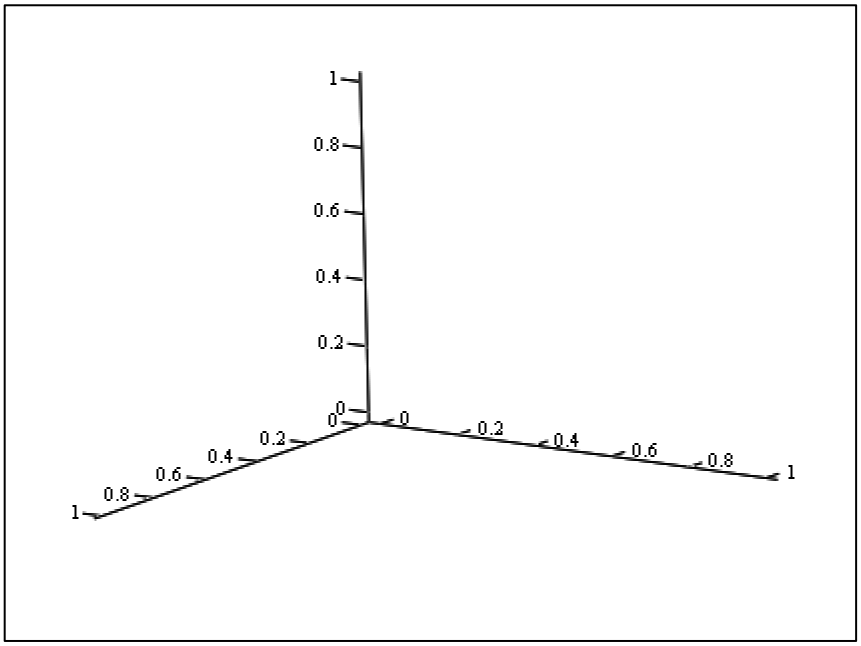
$$x6_{\tau,s} := \frac{1}{2} \quad y6_{\tau,s} := \frac{1}{2} - t(s) \quad z6_{\tau,s} := 0$$ Change the color to blue

In the $-\vec{j}$ direction on the surface

$$x7_{\tau,s} := \frac{1}{2} \quad y7_{\tau,s} := \frac{1}{2} - t(s) \quad z7_{\tau,s} := 1 - \left(x7_{\tau,s}\right)^2 - \left(y7_{\tau,s}\right)^2$$ Change the color to blue

Now Animate -- I used 20 frames at 1 frame/sec. You could add more vectors and are only limited by being able to use up to 16 different plots on a single graph.

See Animation Directional Derivative



(X, Y, Z) , (x , y , z) , (x1, y1 , z1) , (x2, y2 , z2) , (x3, y3 , z3) , (x4, y4 , z4) , (x5, y5 , z5) , (x6, y6 , z6) , (x7, y7 , z7)