Lab 3 Cylindrical Coordinates

You have seen that in 2-space many curves which would be very difficult to analyze and graph in rectangular coordinates are very easy to describe if we use polar coordinates. For example circles have the very simple form r =c. Other examples include cardioids, spirals, limacons, and many petaled "roses".

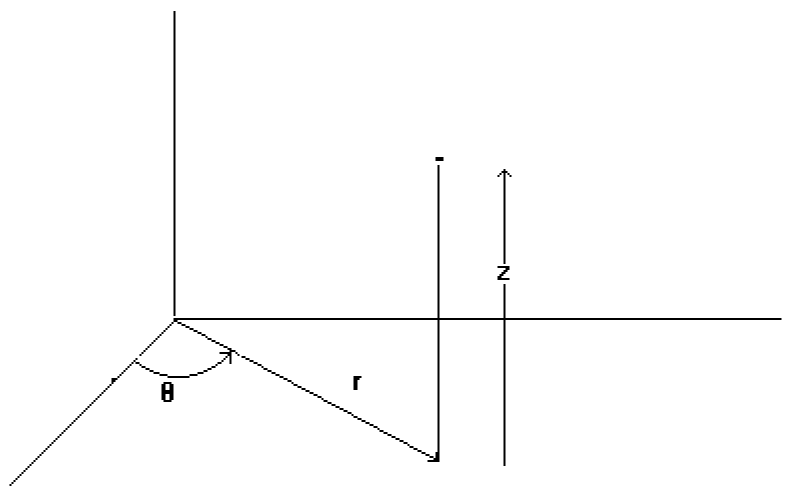
In 3 -space there are two alternative coordinate systems which are used: cylindrical and spherical coordinates. In this lab we will consider cylindrical coordinates

The cylindrical coordinate system is the direct 3-d analog of polar coordinates. We simply add a z component (the same z as in rectangular coordinates) to the r and θ components. Then every point in 3 space can be described by a unique ordered triple (r,θ,z) where :

$0 < \theta < 2\pi$

$0 < r < \infty$

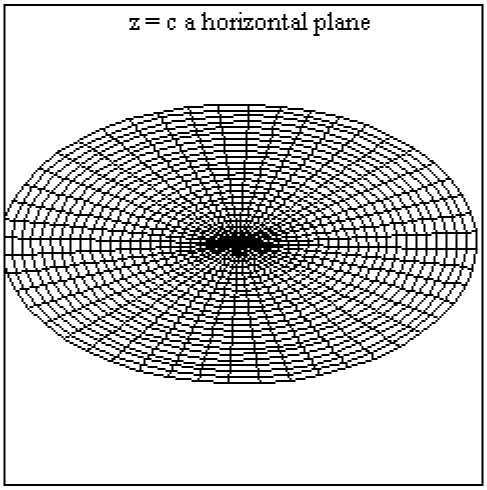
$-\infty < z < \infty$

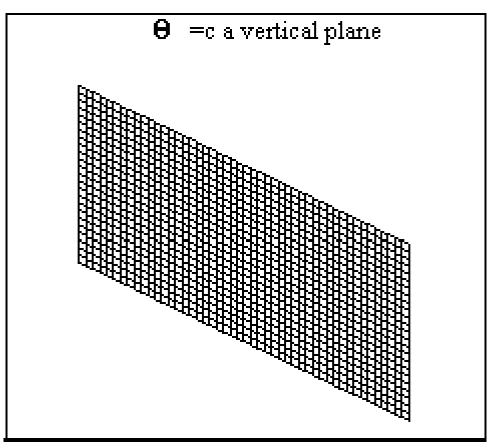


Functions

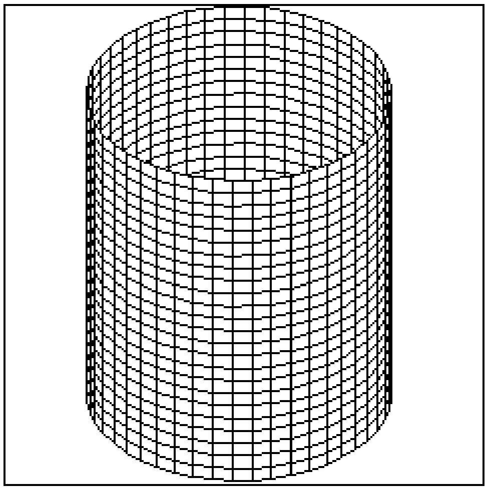
Functions in rectangular coordinates are usually in the form z = f(x,y), in cylindrical coordinates functions are usually of the form z = f(r,θ). As with rectangular coordinates a surface in cylindrical coordinates is the set of all points {r,θ,f(r,θ)}.

Constant functions and relations.





Recall in polar coordinates θ =c is a line. Translating a line in the z direction sweeps out a plane.



r = c a vertical cylinder

Recall in polar coordinates r = c is a circle. Translating a circle in the z direction sweeps out a cylinder

Conversion to Rectangular coordinates

In a previous lab we mentioned that Mathcad can only plot over a rectangular domain.

However, we can plot over various other domains by defining a function in cylindrical coordinates and using the conversion to rectangular coordinates and using what is known as a parametric surface plot. The details are presented below.

But first recall the conversion of x and y to polar coordinates:

$x = r\cos(\theta) \quad y = r\sin(\theta)$

In cylindrical coordinates we have:

$x = r\cos(\theta) \quad y = r\sin(\theta) \quad z = z$

Formatting the computer

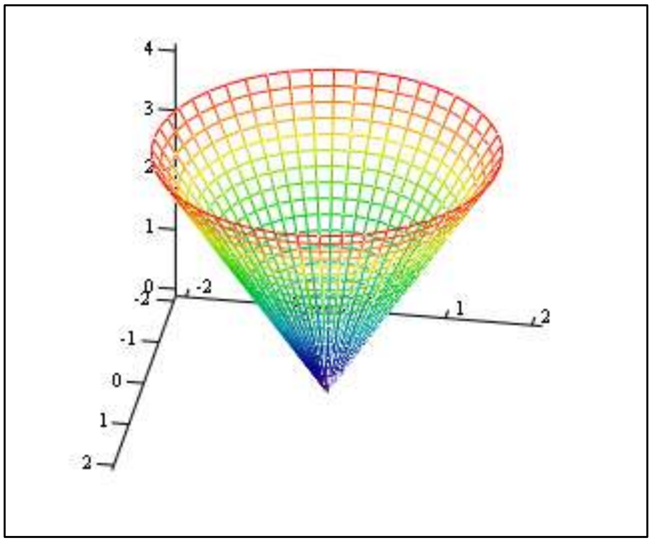
$i := 0..48 \quad \theta_i := \pi \cdot \frac{i}{24}$ This allows θ to vary from 0 to 2π in increments of π/24. This is standard for almost every case (as with polar coordinates). If we want a smaller step let i be large enough so that θ varies from 0 to 2π.

$j := 0..20 \quad r_j := \frac{j}{10}$ This allows r to vary from 0 to 2 in increments of .1. The range on r is dependent on the particular problem. With these definitions we now have a circular domain.

$X_{i,j} := r_j \cdot \cos(\theta_i) \quad Y_{i,j} := r_j \cdot \sin(\theta_i)$ These 2 equations give x and y in terms of cylindrical coordinates.

$Z_{i,j} := 2 \cdot r_j$ Here we would now define our function z = F(r,θ). To illustrate we're using z = 2r, a cone.

We proceed as before i.e. we pull down CREATE SURFACE PLOT. However, this time in the box for array we type :(X,Y,Z) as in the figure below. You Must use Parentheses



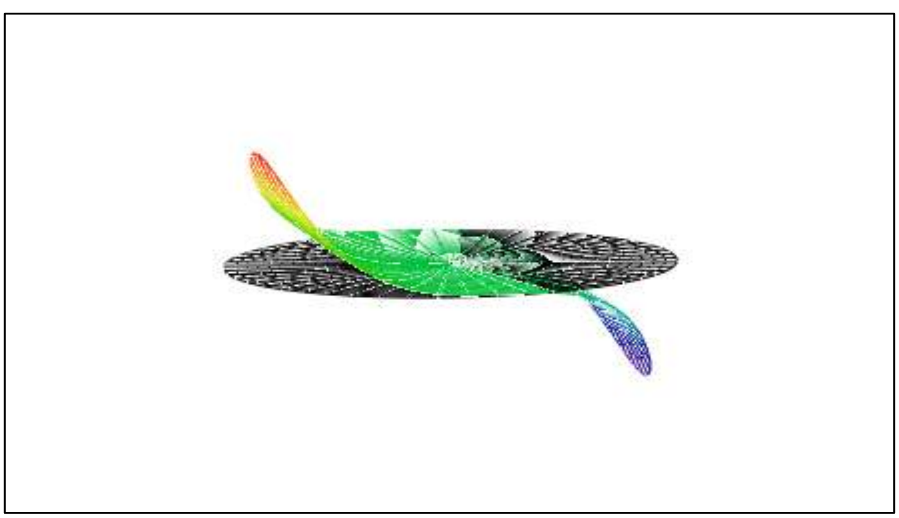
(X, Y, Z)

In this example we graph on an elliptical domain

$i := 0..48$

$\theta_i := \pi \cdot \frac{i}{24}$ $j := 0..20$ $r_j := \frac{j}{10}$

$X_{i,j} := (2 \cdot r)_j \cdot \cos(\theta_i)$ $Y_{i,j} := r_j \cdot \sin(\theta_i)$ $Z_{i,j} := 4 \cdot (X_{i,j})^3 + (Y_{i,j})^2$ $Z2_{i,j} := 0$



(X, Y, Z), (X, Y, Z2)

Let's do one more example before I turn it over to you. Let's graph the paraboloid $z = 1 - x^2 - y^2$

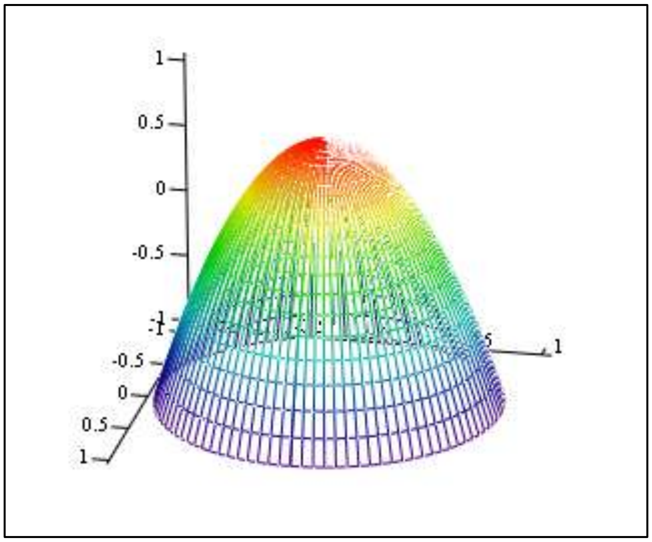
over the unit circle i.e. r = 1. $z = (1 - x^2 - y^2) = [1 - [r^2 \cos^2(\theta)] - r^2 \sin^2(\theta)] = 1 - 2 \cdot r^2$.

Let's use a step size of $\pi/48$

$i := 0..\, 96$ $\theta_i := \pi \cdot \frac{i}{48}$ This allows θ to vary from 0 to 2π in increments of $\pi/48$. This time i goes to 96

$j := 0..\, 20$ $r_j := \frac{j}{20}$ This allows r to vary from 0 to 1 in increments of 0.5.

$X_{i,j} := r_j \cdot \cos(\theta_i)$ $Y_{i,j} := r_j \cdot \sin(\theta_i)$ $Z_{i,j} := 1 - 2 \cdot (r_j)^2$



(X, Y, Z)

Exercises

A. Graph the following functions:

1. **z =e $^{-5r}$**

2. **z = r^2**

3 **z = $r^2\cos(2\theta)$ (consider several rotations)**

Show using the double angle formula for cos(2θ) this is the saddle z = $x^2 - y^2$

B. Cylindrical surfaces in cylindrical coordinates

1. Plot and describe the surface $r = \theta$ (define $Z_{i,j} = j$ and $r_i = \theta_i$). Let j range from 0 to 20 as above and let i range from 0 to 192. *<u>In this particular example r must be subscripted with i.</u>* This is the equivalent of a cylindrical surface and should look like a spiral extended in the z direction

2. Plot and describe the surface $r = 5 - 10\cos(\theta)$. Formatted as above with $r_i = 5 - 10\cos(\theta_i)$.
Again, let $Z_{i,j} = j$. It should look like a limacon extended in the z direction