

### Graphing parametric curves on surfaces

Here we will be using the paraboloid  $z = 1 - x^2 - y^2$  with domain the unit circle  $r = 1$  in the first quadrant

Suppose in the domain we are moving from the origin on the line parameterized by:

$$x(t) := t \quad y(t) := t$$

Then on the surface we induce the parameterization:

$$x(t) := t \quad y(t) := t \quad z(t) := 1 - 2 \cdot t^2$$

We start by defining the surface and putting in the Axes

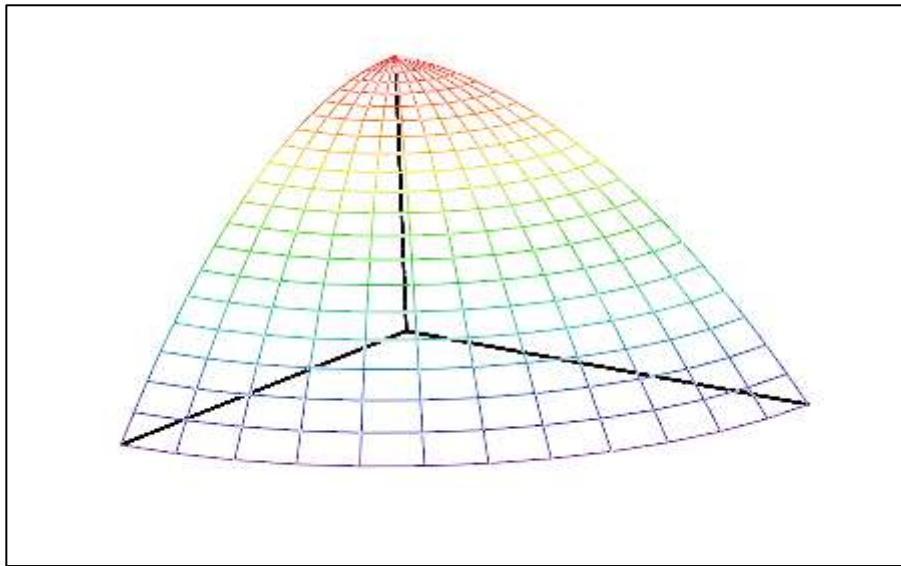
$$i := 0..12 \quad \theta_i := \pi \cdot \frac{i}{24} \quad j := 0..20 \quad r_j := \frac{j}{20}$$

$$X_{i,j} := r_j \cdot \cos(\theta_i) \quad Y_{i,j} := r_j \cdot \sin(\theta_i) \quad Z_{i,j} := 1 - (X_{i,j})^2 - (Y_{i,j})^2$$

Axes:

$$m := 0..10 \quad X1_{i,m} := 0 \quad Y1_{i,m} := 0 \quad Z1_{i,m} := m \cdot 1$$

for Plot 1 change weight to .5 to lighten the surface.



(X, Y, Z), (X1, Y1, Z1), (Y1, Z1, X1), (Z1, X1, Y1)

Now we define the curves

$$s := 0..FRAM \quad t(s) := s \cdot .02 \quad \tau := 0..1$$

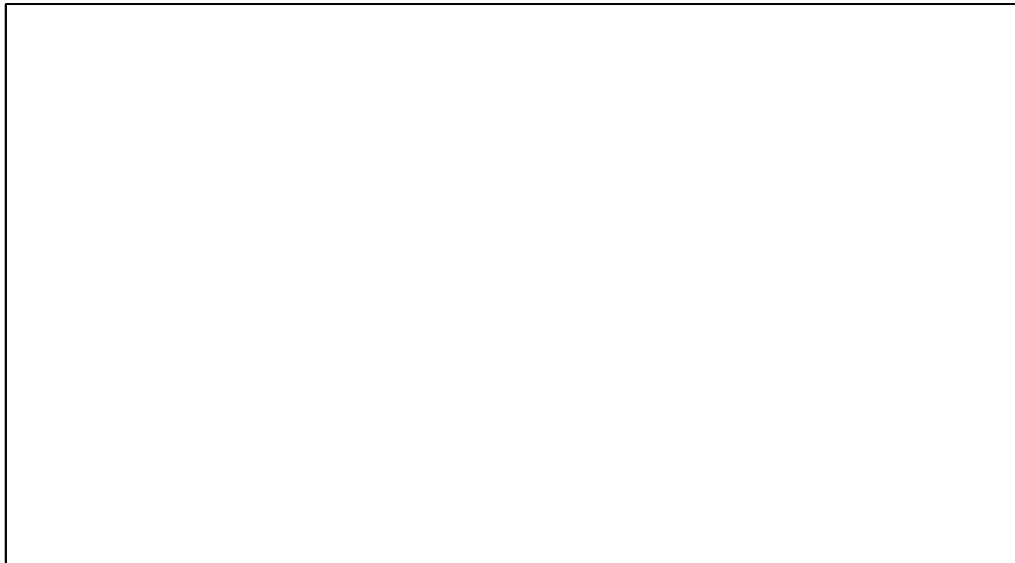
$$\text{Curve in the plane} \quad \begin{matrix} x_{\tau,s} \\ \text{---} \end{matrix} := t(s) \quad \begin{matrix} y_{\tau,s} \\ \text{---} \end{matrix} := t(s) \quad \begin{matrix} z_{\tau,s} \\ \text{---} \end{matrix} := 0$$

$$\text{Curve on the surface} \quad \begin{matrix} x_{\tau,s} \\ \text{---} \end{matrix} := t(s) \quad \begin{matrix} y_{\tau,s} \\ \text{---} \end{matrix} := t(s) \quad \begin{matrix} z_{\tau,s} \\ \text{---} \end{matrix} := 1 - 2 \cdot t(s)^2$$

For Plots 5 and 6 under the appearance tab change weight to 2 to darken the curves

To animate use 35 frames since  $z=0$  when  $1 - 2 \cdot t^2 = 0$  when  $t = \frac{1}{\sqrt{2}} = .7$

$$t(s) = .02s = .7 \quad \text{for } s = 35$$



$(X, Y, Z), (X1, Y1, Z1), (Y1, Z1, X1), (Z1, X1, Y1), (x, y, z), (x, y, z1)$

Suppose in the domain we instead are following the curve  $y = t^2$

$$x(t) := 1 \quad y(t) := t^2$$

Then on the surface we induce the parameterization:

$$\underline{x}(t) := 1 \quad \underline{y}(t) := t^2 \quad z(t) := 1 - t^2 - t^4$$

$$i := 0..12 \quad \theta_i := \pi \cdot \frac{i}{24} \quad j := 0..20 \quad r_j := \frac{j}{20}$$

$$X_{i,j} := r_j \cdot \cos(\theta_i) \quad Y_{i,j} := r_j \cdot \sin(\theta_i) \quad Z_{i,j} := 1 - (X_{i,j})^2 - (Y_{i,j})^2$$

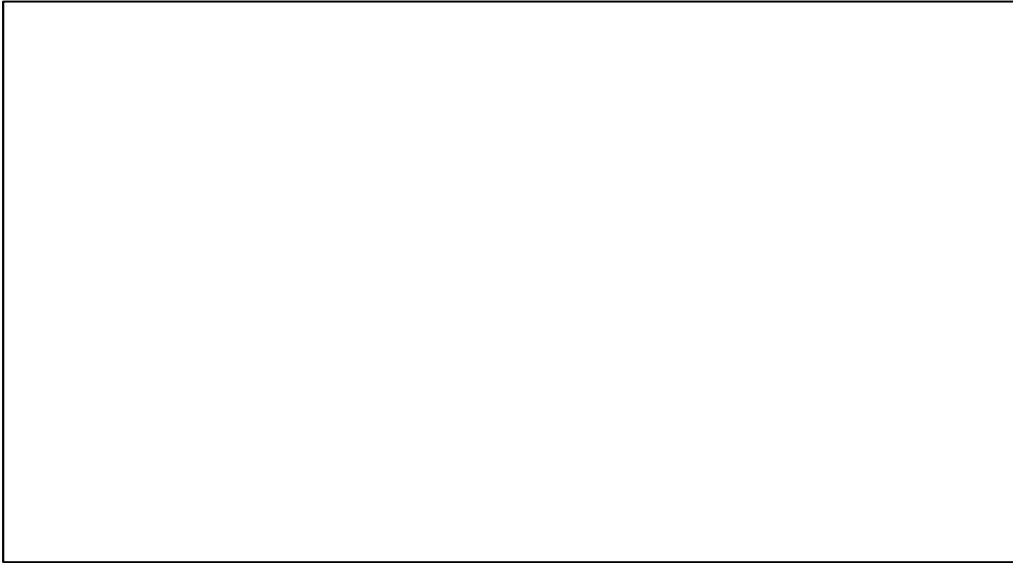
$$m := 0..10 \quad X1_{i,m} := 0 \quad Y1_{i,m} := 0 \quad Z1_{i,m} := m \cdot 1$$

$$s := 0..FRAM\# \quad t(s) := s \cdot .02 \quad \tau := 0..1$$

Curve in the plane  $\underline{x}_{\tau,s} := t(s) \quad \underline{y}_{\tau,s} := t(s)^2 \quad \underline{z}_{\tau,s} := 0$

Curve on the surface  $\underline{x}_{\tau,s} := t(s) \quad \underline{y}_{\tau,s} := t(s)^2 \quad \underline{z}_{\tau,s} := 1 - t(s)^2 - t(s)^4$

To animate use 39 frames since  $z=0$  when  $1 - 2 \cdot t^2 = 0$  when  $t = \frac{1}{\sqrt{2}} = .707$   
 $t(s) = .02s = .707$  for  $s = 35.3$



**(X, Y, Z), (X1, Y1, Z1), (Y1, Z1, X1), (Z1, X1, Y1), (x, y, z), (x, y, z1)**